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Cellular automaton model for railway traffic

KePing Li *, ZiYou Gao, Bin Ning

School of Traffic and Transportation, Beijing Jiaotong University, 3 Shangyuancun, Xizhimenwai, Beijing 100044, PR China

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Abstract

Based on the deterministic NaSch model, we propose a new cellular automation (CA) model to simulate the railway traffic. Using the proposed CA model, we analyze the space–time diagram of traffic flow and the trajectory of train movement, etc. Our aim is to investigate the characteristic behavior of railway traffic flow. A number of simulation results demonstrate that the proposed model can be successfully used for the simulation of railway traffic. Some complex phenomena can be reproduced, such as the go-and-stop wave and the complex behavior of train movement.

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Keywords: Cellular automation; Traffic flow; Moving-block system; Fixed-block system

1. Introduction

A few decades ago, moving-block signalling system was proposed to provide more room for headway reduction [1]. In this system, its operations rely on continuous bi-directional communication links between trains and controllers. However, most successful implementations of moving-block systems are not exactly utilizing the concept in its original form [2,3]. The communication is not absolutely continuous. As signalling technology developed, moving-block system has gained considerable importance and there have been on attempts to instead fixed-block system. It offers the advantage to allow the distance between two trains to be varied according to their actual speeds. This is rather like applying the freeway rules for speed separation. In theory, the trains may get as close as a few meters, just sufficient allowance for reaction time and small errors.

* Corresponding author.
E-mail address: kpli@eyou.com (K. Li).

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NaSch model is one of the microscopic traffic models [4]. Using very simple rules, this model can reproduce the basic phenomena encountered in real traffic. The advantage of NaSch model is that it is much simpler and more convenient for computer simulations (it is able to perform several millions updates in a second [5]). For the description of more complex situations, the basic rules of NaSch model have to be modified, such as the multi-lane traffic, bidirectional traffic and the traffic with different types of vehicles, etc. [6–9].

Computer simulation can be used to test theoretical control algorithms for improving the performance of railway signalling system. Several types of models have been developed for the train movement calculation, such as the basic model, the time-based model and the event-based model, etc. [10,11]. Using the basic model, when the length of track conductor loop is small, a continuous braking profile is a reasonable approximation. The time-based approach is easy to design and build simulation models, but it makes a high computational demand. The event-based model substantially saves computational effort, but inevitably at the expense of a certain degree of accuracy. In this work, we propose a new cellular automation (CA) traffic model for simulating the railway traffic. Our model has the following features: (1) Using very simple rules, our model can be used to simulate the railway traffic with shorter computation time. (2) It is a flexible model, and is easy to simulate different types of railway traffic by modifying the basic rules of the model. To our knowledge, this work explicitly shows this effect for cellular automata for the first time. The paper is organized as follows: we introduce the principle of the railway signalling system in Section 2, and introduce the simulation model in Section 3; the numerical and analytical results are presented in Section 4; finally, conclusion of this approach is presented.

2. Principle of the railway signalling system

2.1. Moving-block system

In a moving-block system, the traditional fixed-block track circuit is not necessary for determining the train position. Instead, the continuous two-way digital communication between each controlled train and a wayside control center are adopted. In this system, the line is usually divided into areas or regions, each area is under the control of a computer and has own radio transmission system. Each train transmits its identity, location, direction and speed to the area computer. The radio link between each train and the area computer is continuous so that the computer knows the location of all the trains in its area all the time. It transmits to each train the location of the train in front and gives it a braking curve to enable it to stop before it reaches that train. In theory, as long as each train is travelling at the same speed as the one in front and they all have the same braking capabilities, they can travel as close together as a few meters. The system adopted in London Dockland light rail (DLR) is very well-documented [2]. In this system, the safety–critical or 'vitally programmed' computers and high-integrity telemetry were used to maintain continuous communication between the control center and trains. A similar approach has been found in the LZB510 system [12], in which track conductor loops are used to improve the performance of train control.

Several types of moving-block scheme have been discussed [1]. Moving space block (MSB) is the simplest scheme, in which the minimum instantaneous distance between successive trains is

$$d_n = v_{\max}^2/(2b) + SM, \tag{1}$$

where $v_{\max}$ is the maximum speed, b denotes the deceleration of the train n and SM is the safety margin distance. In this scheme, the only information that the following train requires is the position of the leading train and its own position.

Moving time block (MTB) is another scheme, in which the headway between two trains is always constant and independent of travel speed. In this scheme, the minimum instantaneous distance between successive trains is

$$d_n = v_{\max} v_{\mathrm{f}}/(2b) + SM, \tag{2}$$

where v_{f} is the speed of the following train.

Pure moving-block (PMB) is also a type of moving-block system, in which the requirement information is the relative position of successive trains and the speed of the following train. In this scheme, the minimum instantaneous distance between successive trains is

$$d_n = v_{\mathrm{f}}^2/(2b) + SM, \tag{3}$$

where the distance is basically the braking distance required by a train, and is the minimum distance needed to avoid a collision even if the leading train comes to a sudden halt. This latter system gives the best performance and is the basis of all currently implemented systems. Fig. 1(a) indicates the principle of the moving-block signalling system. In Fig. 1(a), the difference of the three types of moving-block (MSB, MTB, PMB) is the minimum instantaneous distance between successive trains.

On a moving-block equipped railway, suppose the leading train has occupied the station, and the following train n is behind the station, the minimum instantaneous distance between the train n and the station is as follow:

$$d_n = SM + v_{\max}^2/(2b), \tag{4}$$

here the first term in the right-hand side represents the safety margin distance, and the second term in the right-hand side represents the braking distance from the maximum speed $v_{\max}$ to a stop.

2.2. Fixed-block system

Fixed-block signalling (FBS) is another system in railway traffic, it has been widely used on modern railways for more than a century. In the FBS scheme, the line is divided into blocks. The length of blocks depends on the maximum speed, the braking rate and the number of signalling aspects. At any time, one block of track can only been occupied by no more than one train. This feature of train movement is controlled by the signalling light. In general, when the color of signalling light in front of train n is red, the train n does not allowed to move into the leading block, if the color of signalling light in front of train n is yellow, the train n can move with

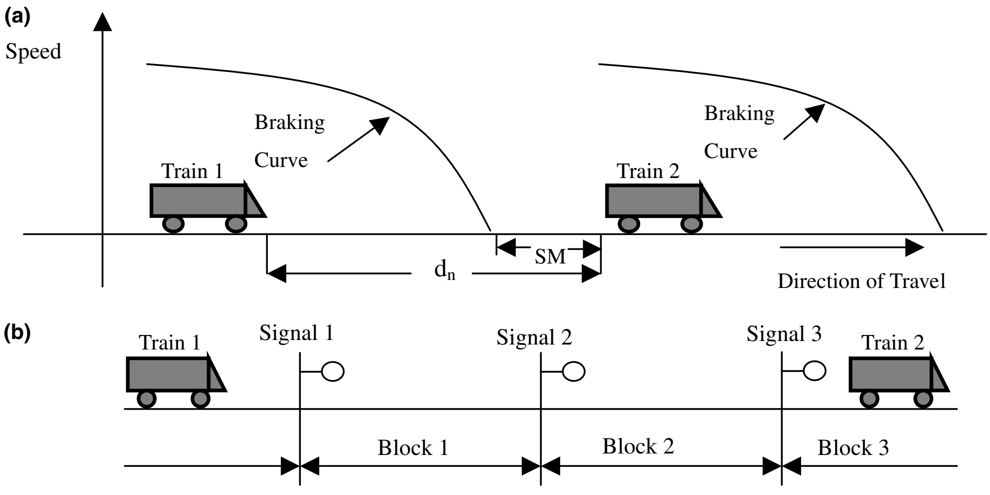

Fig. 1. (a) Principle of the moving-block signalling system. Train 1 is the following train and train 2 is the leading train. In moving-block system, when the distance headway between the train 1 and train 2 is smaller than the minimum instantaneous distance d_n, the train 1 must decrease to a lower speed. (b) Principle of the fixed-block signalling system.

a limited speed, otherwise it can move with a larger speed (i.e., the color of signalling light in front of train n is green). Fig. 1(b) indicates the principle of the fixed-block signalling system. Here the signalling aspect is three-aspect. In Fig. 1(b), the signalling light has three colors, i.e., red, yellow and green. As shown in Fig. 1(b), the train 2 is in block 3, the color of signalling light 3 is red, the color of signalling light 2 is yellow and the color of signalling light 1 is green. In this case, the train 1 can move into the block 1 with a larger speed.

3. The simulation model

3.1. Cellular automata model

One of the traffic models is the CA model, which was introduced in the 1950s by von Neumann [13]. However, it received the attention of a wider audience in the 1970s through Conway's game of life [14]. The first CA model for vehicular traffic was introduced by Cremer and Ludwig [15]. In the CA models of traffic, the position, speed, acceleration as well as time are treated as discrete variables. In this approach, a lane is represented by a one-dimensional lattice. Each of the lattice sites represents a cell which can be either empty or occupied by at most one train at a given instant of time. At each discrete time step $t \to t+1$, the state of the system is updated following a well defined prescription.

Nagel and Schreckenberg developed a one-dimensional probabilistic CA model which is a model of traffic flow on a single lane [4]. In NaSch model, the road is divided into L cells numbered by $i = 1, 2, \ldots, L$, and the time is discrete. Each site can be either empty or occupied by a vehicle with integer speed $v = 0, 1, \ldots, v_{\max}$. The underlying dynamics of NaSch model is governed by the updated rules applied at discrete time steps. All sites are simultaneously updated according to four successive steps: (1) acceleration: increase v_n by 1 if $v_n < v_{\max}$; (2) slowing down: decrease v_n to $v_n = d$ if necessary (d is the number of empty cells in front of the vehicle); (3) randomization: decrease v_n by 1 with randomization probability p if $p > 0$; (4) movement: move vehicle v_n sites forward. Here either period boundary condition or open boundary condition can be considered.

3.2. The proposed CA model

In this work, we use CA model to simulate the railway traffic with the moving-block signalling system. Our investigation is based on the NaSch traffic model [4]. The lane consists of a single lane which is divided into L cells of equal size numbered by $i = 1, 2, \ldots, L$, and the time is discrete. Each site can be either empty or occupied by a train with integer speed $v_n = 0, 1, \ldots, v_{\max}$.

In railway traffic, in order to avoid the collision between two trains, the distance between the two trains must be larger than the minimum instantaneous distance. For this purpose, in our method, the acceleration of the nth train is given by the following step:

if $\Delta x_n > d_n$
  $v_n = \min(v_n + a, v_{\max})$.
elseif $\Delta x_n < d_n$
  $v_n = \max(v_n - b, 0)$.
else
  $v_n = v_n$
end

where Δx_n is the distance headway and a is the acceleration of the nth train. When the headway Δx_n is larger than the minimum instantaneous distance d_n, the nth train is accelerated by a. If the headway Δx_n is less than the minimum instantaneous distance d_n, the nth train is decelerated by b.

When the nth train is directly behind a station, if the station is occupied by a train, the nth train must keep the minimum instantaneous distance from the leading train, and if the station is empty, the nth train enter the station directly. After the station dwell time T_d, the train leaves the station. When the station is empty, the acceleration of the nth train is given by the following step:

if $x_s > x_c$
  $v_n = \min(v_n + a, v_{\max})$.
elseif $x_s < x_c$
  $v_n = \max(v_n - b, 0)$.
else
  $v_n = v_n$
end

where x_s is the distance from the nth train to the station in front of it, and x_c is the distance that the nth train can enter the station by deceleration.

Compared our model to NaSch model, step 3 adopted in NaSch model is ignored, i.e., the randomization probability p is $p = 0$. The update rules for implementing the railway traffic are as follows:

Case I: The train n is behind the train $(n - 1)$
Step 1 acceleration:
if $\Delta x_n > d_n$
  $v_n = \min(v_n + a, v_{\max})$
elseif $\Delta x_n < d_n$
  $v_n = \max(v_n - b, 0)$
else
  $v_n = v_n$
end
Step 2 slowing down:
  $v_n = \min(v_n, gap)$
Step 3 movement:
  $x_n = x_n + v_n$
Case II: The train n is behind a station.
(1) The station is occupied by a train, the update rules are same as that used in Case I.
(2) The station is empty.
Step 1 acceleration:
if $x_s > x_c$
  $v_n = \min(v_n + a, v_{\max})$.
elseif $x_s < x_c$
  $v_n = \max(v_n - b, 0)$.
else
  $v_n = v_n$
end
Step 2 slowing down:
  $v_n = \min(v_n, gap)$
Step 3 movement:
  $x_n = x_n + v_n$

The update scheme of the CA model is illustrated with a simple example in Fig. 2. In our method, the boundary condition for CA model is open. It is defined as: (1) when the section from the site 1 to the site L_s is empty, a train with the speed $v_{\max}$ is created. This train immediately moves according to the update rules. (2) at site $i = L$, the trains simply move out of the system. In order to compare simulation results to field measurements, one CA iteration roughly corresponds to 1 s, and the length of a cell is about 1 m. This means, for example, that $v_{\max} = 10$ cells/update corresponds to $v_{\max} = 36$ km/h.

4. Simulations

4.1. Modelling moving-block system

We use the CA model proposed in this paper to simulate the railway traffic. A system of $L = 1000$ is considered, and the length of evolution time is $T = 1000$. One station is designated at the middle of system. The moving-block scheme adopted in this work is MSB. The parameters a and b are all set to be 1.

In railway traffic with the moving-block signalling system, some complex phenomena that are found in road traffic system can also be observed, such as the go-and-stop waves. Fig. 3 shows the space–time diagram of traffic flow. Here we plot 1000 sites in 500 consecutive time steps. The abscissa indicates the direction of train travel, and the ordinate indicates time. In Fig. 3, the positions of trains are indicated by dots. From Fig. 3, we can see that the traffic flow in front of the station is free, where trains can travel freely,

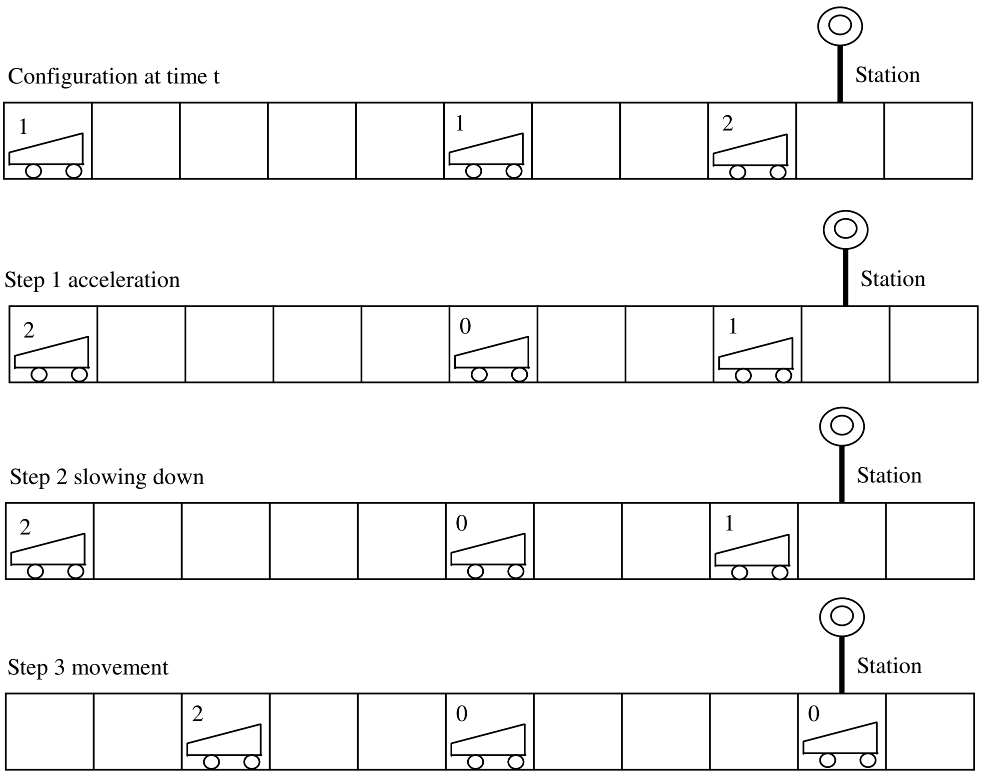

Fig. 2. Step-by-step example for the application of the update rules. We have assumed $v_{\max} = 5$ and $d_n = 4$. The number in the upper left corner is the speed of the train.

however, it is synchronized flow behind the station. These traffic phases alternatively appear and exhibit strip-like distribution in the evolution of traffic flow. These are so-called go-and-stop waves which propagate backward.

The formation of go-and-stop waves is related to many factors, such as the train density ρ, the station dwell time T_{d} and the minimum instantaneous distance d_n, etc. One of the important factors is the train density ρ. The computer simulation results demonstrate that when the density ρ is lower, the traffic flow is free, with the density ρ increased, the go-and-stop waves begin to emerge. Within the go-and-stop waves, the distance headway between following trains are not the same for all stopped trains. The reason is that when the distance between two successive trains is smaller than the minimum instantaneous distance, the following train would be forced to brake to a lower speed. The decreased-effect leads to the formation of the go-and-stop waves. In general, when the following trains begin to decrease, their speeds and distance headway are not the same so that the distance headway within the go-and-stop waves are different for all stopped trains. In this work, the parameter L_s determines the train density ρ, and it can be chosen suitably. The smaller the parameter L_s is, the higher the train density ρ is.

Fig. 4 presents the local space–time diagram, which displays the positions and speeds of the trains that are within the go-and-stop waves. Here numbers represent the speeds of trains, and dots correspond to empty sites. From Fig. 4, we can clearly see that when the train 1 is delayed, its speed will be decreased, at the next step, the train 2 that is directly behind the train 1 will also be decreased. As the time proceeds, a number of trains that are behind the train 1 will be delayed. This is the reason that the quasi-jams observed in Fig. 3 form. From the practical views, the reason of the train delay is that: (1) the train is at the station within the station dwell time T_{d}; (2) the train density ρ is larger.

During the simulations, if the distance between two successive trains is smaller than the minimum instantaneous distance, the trains will interact through the control signalling, and the following train will be forced to brake to a lower speed or stop at a site. The trajectory of one train is shown in Fig. 5. It displays the position and time of the train. Here the train is tracked at the time $t = 600$. In Fig. 5, the horizontal denotes that the train stop at a site of the railway, where its speed is zero. From Fig. 5, it is obvious that there are several horizontal lines, one is about the station dwell, and the other are about the delay of trains within the go-and-stop waves.

When the station is occupied by a train, the train which is directly behind the station must keep away from the station. If the station is empty, this train will enter the station by decreasing its speed. The dynamic response of one train which is directly behind the station is shown in Fig. 6. Here the solid line

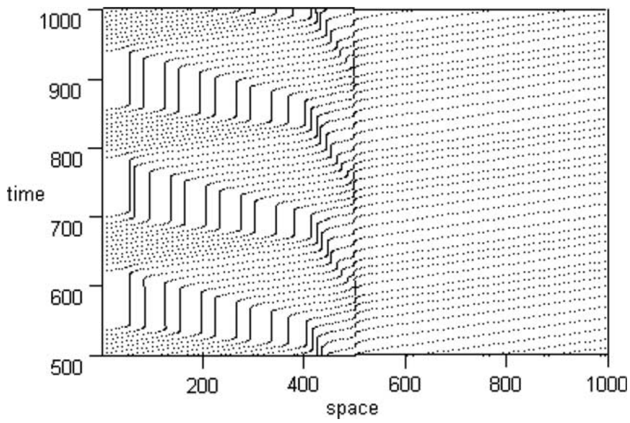

Fig. 3. Local space–time diagram of traffic flow for $v_{\max} = 10$, $L_s = 60$, $SM = 10$ and $T_{\mathrm{d}} = 5$. The site $l > 500$ is in front of the station. The site $l < 500$ is behind the station.

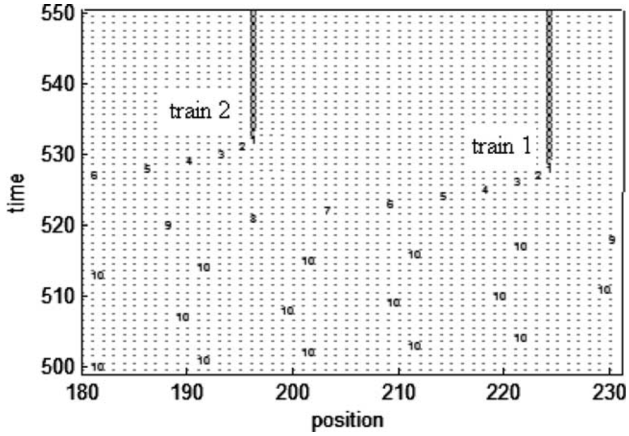

Fig. 4. A diagram displaying the positions and speeds of the trains for $v_{\max} = 10$, $L_s = 60$, $SM = 10$ and $T_d = 5$. Trains 1 and 2 are delayed, they decrease from the maximum speed $v_{\max}$ to zero speed.

denotes the speed for $T_d = 5$, and the dotted line denotes the speed for $T_d = 10$. In Fig. 6, this train is tracked at the time $t = 600$. From Fig. 6, we can see that this train travel toward the station with the maximum speed $v_{\max}$, when the distance between the train and the station is smaller than the parameter x_c, this train will decelerate until it stop at the station. After the station dwell time T_d, it will accelerate gradually, and then leave the station. In general, when the dwell time T_d is larger, the speed variance is more complicated.

In railway traffic, it is very important to take into account the safety factor. This means that the distance between two successive trains must be larger than (or equal to) the minimum instantaneous distance. In our study, the numerical simulation results show that within the go-and-stop waves or near the station, the distance between two successive trains is smaller than the minimum instantaneous distance, however, it is larger than the minimum instantaneous distance out of these regions. In order to investigate this character of the railway traffic flow, we measure the distribution of distance headway. $\{h_i\}$, $i = 1, \ldots, M$, is recorded as the distance headway at a given time t, where h_i is the distance from the ith train to the $(i + 1)$th train. Fig. 7 shows the distribution of the distance-headway at the time $t = 1000$. The minimum

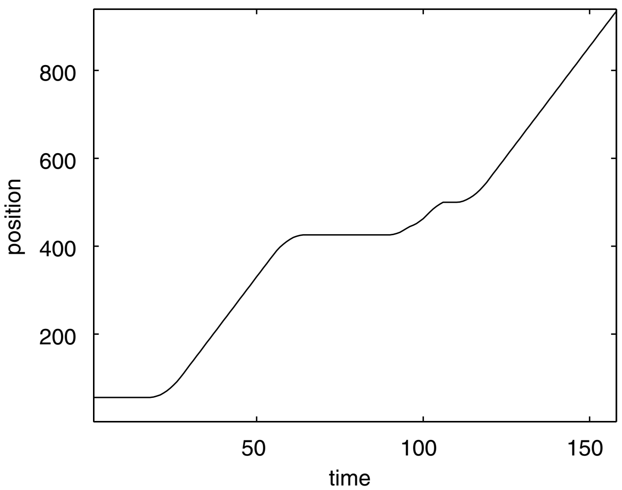

Fig. 5. Trajectory of one train for $v_{\max} = 10$, $L_s = 60$, $SM = 10$ and $T_d = 5$. It takes 160 time steps that the train travel from the site $i = 1$ to the site $i = 1000$.

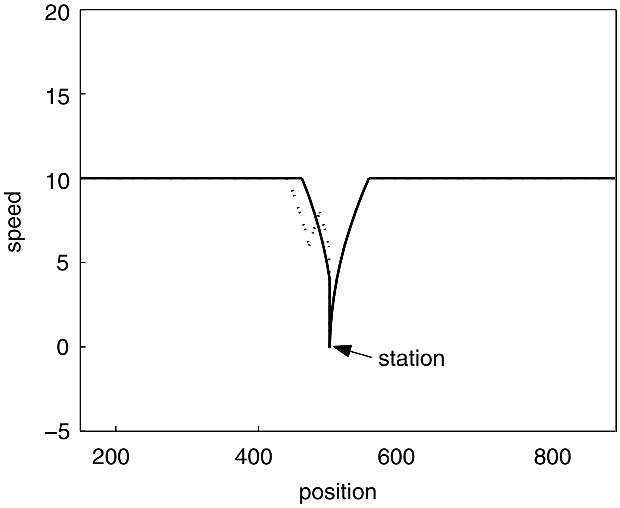

Fig. 6. A diagram displaying the position and speed of one train for $v_{\max} = 10$, $L_s = 150$ and $SM = 10$. During the train entering the station, when the dwell time is $T_d = 5$, it decreases gradually, however when the dwell time is $T_d = 10$, the train must accelerate again.

instantaneous distance is $d_n = 60$. From Fig. 7, we can see that the trains which near the station are delayed within the dwell time, their distance headway is smaller than the minimum instantaneous distance.

In order to avoid collision, between two successive trains, the following train needs to adjust its speed continuously. Fig. 8 shows the speed pattern of the following train that decreased by the leading train. Here the following train is tracked at the time $t = 600$. In Fig. 8, the following train decreases or accelerates for several times according to the position and speed of the leading train. This simulation result demonstrates that the speed controlling simulated in this work is close to actual controlling by the driver. That is to say, simulating train operation like human drivers who operate actual train. The similar behavior of train can be found in Fig. 6, where the train enters the station by decreasing its speed.

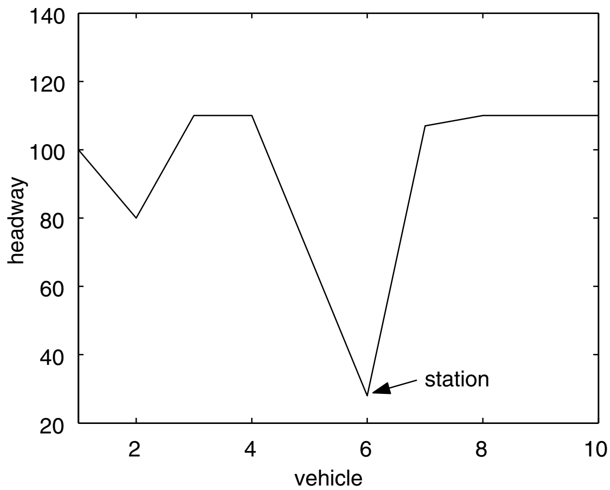

Fig. 7. Distribution of the distance headway for $v_{\max} = 10$, $L_s = 100$, $SM = 10$ and $T_d = 5$. The distance headway near the station is smaller than the minimum instantaneous distance.

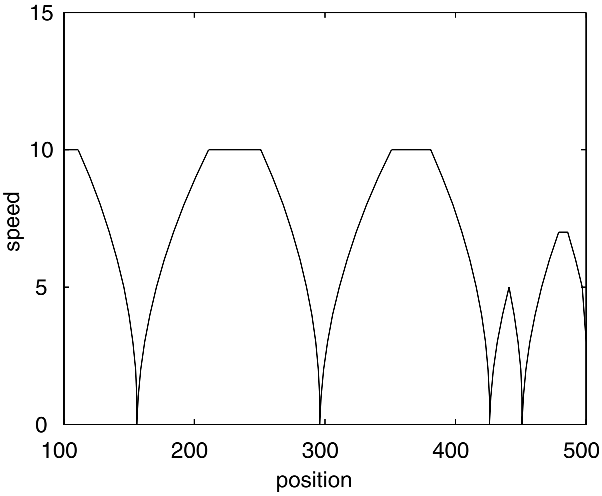

Fig. 8. A diagram displaying the position and speed of one train for $v_{\max} = 10$, $L_s = 100$, $SM = 10$ and $T_d = 8$. In order to avoid collision between two successive trains, the following train adjusts its speed continuously.

In railway system, the most critical period occurs at the approach to station, where the leading train must leave in time for the following train to run into the station following its worst-case braking curve. In this case, the minimum headway time between the two trains is [16]

$$\tau_{\min} = T_r + T_d + \frac{v_{\max}}{b} + \sqrt{\frac{2(L_d + L_t)}{a}}, \tag{5}$$

here T_r is the driver and train equipment reaction time, L_t is the length of the train and L_d is the distance that the leading train has travelled from the station. When the length L_t and the distance L_d are omitted, i.e., $L_t = 0$, $L_d = 0$, the minimum headway time is written as follow:

$$\tau_{\min} = T_r + T_d + \frac{v_{\max}}{b}. \tag{6}$$

In our model, we omit the length L_t and the distance L_d. Using our model to simulate the moving-block signalling system, we record the minimum headway time between two successive trains which are near the station. During the simulations, in order to obtain the time $\tau_{\min}$, the station dwell time T_d is set to be large enough. Fig. 9 shows how the minimum headway time $\tau_{\min}$ varies with the maximum speed $v_{\max}$. From Fig. 9, we can see that the simulation values are close to the theoretical values. This means that the proposed model can be successfully used for the simulation of railway traffic.

4.2. Modelling fixed-block system

As mentioned in the Introduction, one of the advantages of CA model is that it is a flexible model. By modifying the basic rules of CA model, it can be used to simulate different types of railway traffics. As an example, we simulate the railway traffic with three-aspect fixed-block system.

According to the principle of the three-aspect fixed-block signalling system shown in Fig. 1(b), we outline a new CA traffic model for the fixed-block railway system. The update rules for implementing the fixed-block system are as follows:

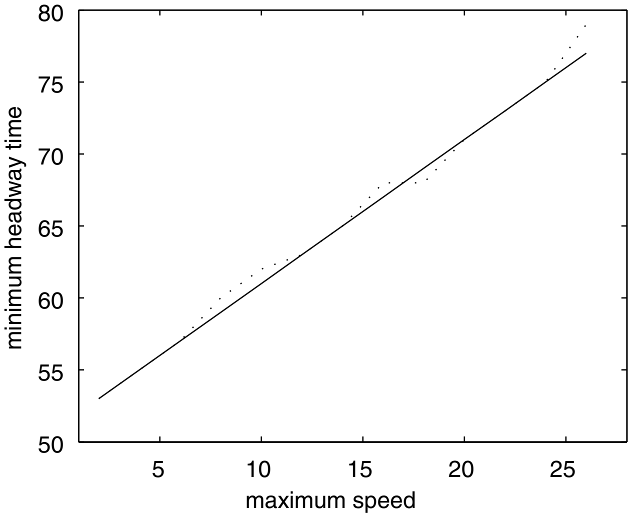

Fig. 9. How the minimum headway time varies with the maximum speed for $T_r = 1$ and $T_d = 50$. The dotted line denotes the simulation values using the proposed model, and the solid line denotes the theoretical values obtained by Eq. (6).

Case I: The color of signalling light is red in front of the nth train
Step 1 acceleration:
if $\Delta x_n > d_b$
  $v_n = \min(v_n + a, v_l)$.
elseif $\Delta x_n < d_b$
  $v_n = \max(v_n - b, 0)$.
else
  $v_n = v_n$
end
Step 2 slowing down:
  $v_n = \min(v_n, gap)$
Step 3 movement:
  $x_n = x_n + v_n$
Case II: The color of signalling light is yellow in front of the nth train
Step 1 acceleration:
  $v_n = \min(v_n + a, v_l)$.
Step 2 movement:
  $x_n = x_n + v_n$
Case III: The color of signalling light is green in front of the nth train
Step 1 acceleration:
  $v_n = \min(v_n + a, v_{\max})$.
Step 2 movement:
  $x_n = x_n + v_n$

where gap and Δx_n are, respectively, the number of empty cells and the distance from the nth train to the closest signalling light in front of the nth train, v_l is the limited speed and d_b is the braking distance. In the modified CA model, the rules for the station and the right boundary condition are same as that proposed in

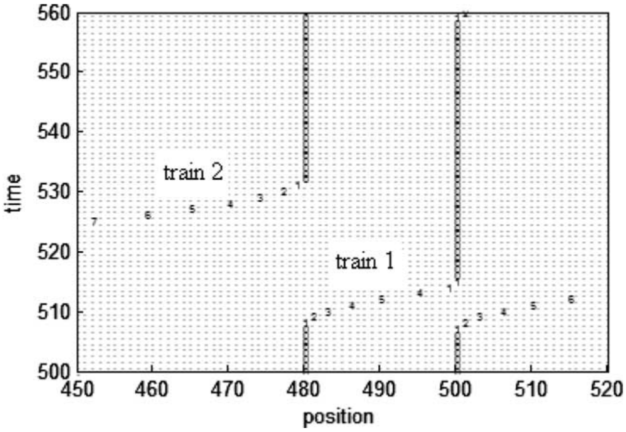

Fig. 10. A diagram displaying the positions and speeds of the trains for $v_{\max} = 10$ and $T_d = 45$. Train 1 and train 2 are the trains near the station, they are delayed.

Section 3.2. The left boundary condition can be defined as: at site $i = 1$, if the first block is empty and the first signalling light is green, a train with the speed $v = v_{\max}$ is created.

We use the modified CA model to simulate the three-aspect fixed-block system. Here the system is divided into 10 blocks. The limited speed is set to be $v_l = 5$. Fig. 10 presents the local space–time diagram, which displays the positions and speeds of the trains near the station (i.e., the site $i = 500$). From Fig. 10, we can clearly see that when the station is occupied by the train 1, the train 2 will be delayed, it will decrease its speed, and finally stops at a site behind the station. After the train 1 leaves the station, the train 2 will accelerate, and then decrease again, until it enter the station. In this case, the trains which are behind the train 2 will also be delayed. These are so-called go-and-stop waves. For a fixed-block signalling system, one of the important factors that are related to the formation of the go-and-stop waves is the station dwell time T_d. The computer simulation results demonstrate that when the station dwell time is smaller, the traffic flow is free, with the station dwell time increased, the go-and-stop waves begin to emerge.

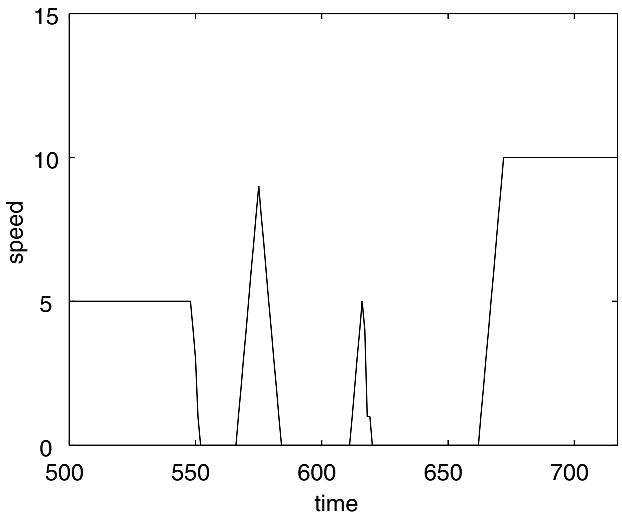

Fig. 11. A diagram displaying the speed and time of one train for $v_{\max} = 10$ and $T_d = 45$.

With a fixed-block signalling system, at any time, only one train may occupy a physical block. If the color of the signalling light in front of the train n is red, this train does not allow to move into its leading block. For a three-aspect fixed-block system, when the number of blocks between two successive trains is less than two, the trains will interact through the signalling light, and the following train will be forced to brake to a lower speed. Fig. 11 shows the speed pattern of one train. Here this train is tracked at the time $t = 500$. In Fig. 11, the train decreases or accelerates for several times. Sometimes the train moves with a limited speed, i.e., $v_n = 5$, sometimes the train moves with a larger speed, i.e., $v_n = 10$, and sometimes the train stops at the line. This simulation result also demonstrates that the speed controlling simulated in this work is close to actual controlling by the driver. That is to say, simulating train operation like human drivers who operate actual train.

5. Conclusions

In conclusions, a new CA model has been proposed to simulate the railway traffic, it is based on the deterministic NaSch traffic model. The proposed model is a basic model, it can be generalized to more complicated traffic conditions by modifying the basic rules, for example, the modelling of mixed traffic. By modifying the CA model suggested in [7], we can simulate the railway system with slow and fast trains, i.e., trains with different maximum speed $v_{\max}$. Similar CA models include the bidirectional traffic model and the traffic networks model, etc. [8,9].

Train movement is not only under the speed restriction, but also is under the constraints of track geometry, traction equipment and train length, etc. In our model, these factors have not been taken into account. This leads that the detail of train movement is omitted in the calculation. As a result, the train movement accuracy, to a certain degree, is reduced. However, the simulation results show that using our method, some characteristic behavior of train movement can be observed. So we think that it is worthy to study further.

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